We study the relationship between the multiplicity of a fixed point of a function \( g \), and the dependence on \( \varepsilon \) of the length of \( \varepsilon \)-neighborhood of any orbit of \( g \), tending to the fixed point. The relationship between these two notions was discovered in [1] in the differentiable case, and related to the box dimension of the orbit.

Here, we generalize these results to non-differentiable cases. We study the space of functions having a development in a Chebyshev scale and use multiplicity with respect to this space of functions. Introducing a new notion of critical Minkowski order, we recover the relationship between multiplicity of fixed points and the dependence on \( \varepsilon \) of the length of \( \varepsilon \)-neighborhoods of orbits in non-differentiable cases where results from [1] do not apply.

Applications include in particular Poincaré map near homoclinic loop and Abelian integrals.

MSC2010: 37G15, 34C05.

Keywords: limit cycles, multiplicity, cyclicity, Chebyshev scale, critical Minkowski order, box dimension, homoclinic loop.

References


[2] P. Mardešić, M. Resman, V. Županović, Multiplicity of fixed points and growth of \( \varepsilon \)-neighborhoods of orbits, preprint

Section: 12.