The coding-theoretical interest in combinatorial designs defined by subspaces of a finite geometry was motivated in the 1960’s by their use for the construction of majority-logic decodable codes. In 1973, Noboru Hamada computed the ranks of the incidence matrices of finite geometry designs over the underlying finite field and made the conjecture that geometric designs have minimum rank among all designs with the given parameters. In all proved cases of the conjecture, the geometric designs not only have minimum rank, but are also the unique designs of minimum rank. Until recently, only a handful of non-geometric designs were known that share the same rank with geometric designs. The lectures discuss some recently discovered infinite families of non-geometric designs that have the same parameters and the same rank as certain geometric designs.