Dodatak - Primjeri prostornih krivulja

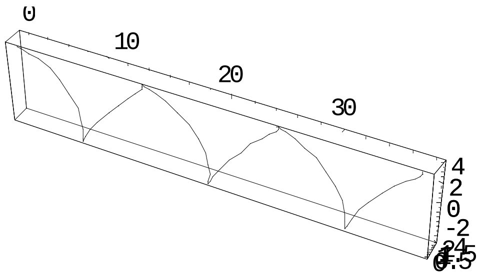
Slijedi prikaz nekih krivulja u prostoru $\mathbb{R}^3$ dobivenih programom *Mathematica*.

U nastavku će se podrazumijevati da je vektorska funkcija $\vec{x}: \mathrm{I} \to \mathbb{R}^3$, $\vec{x} = \vec{x}(t)$, $t \in \mathrm{I}$ zadana sa tri skalarne funkcije $x(t)$, $y(t)$ i $z(t)$ izrazom: $\vec{x}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$,
gdje je $\{\vec{i}, \vec{j}, \vec{k}\}$ ortonormirana baza prostoru $\mathbb{R}^3$.

(1) Vektorska jednadžba:

$$\vec{x} = (t - \sin t)\,\vec{i} + (1 - \cos t)\,\vec{j} + 4\cos\frac{t}{2}\,\vec{k} = \left(t - \sin t, 1 - \cos t, 4\cos\frac{t}{2}\right), \quad t \in [0, 12\pi]$$

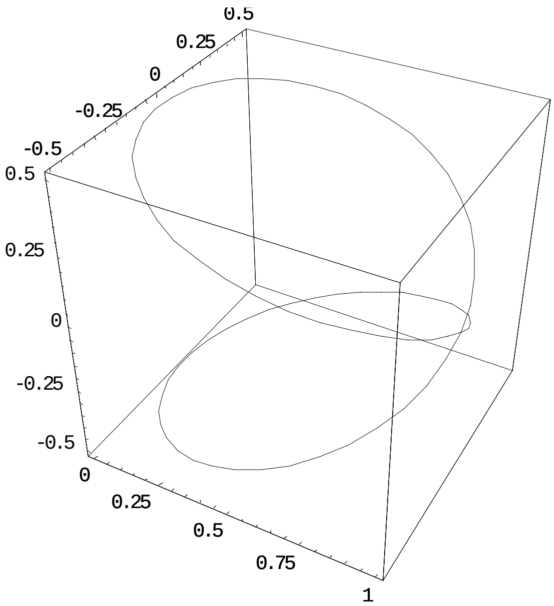
Parametarske jednadžbe: $x = t - \sin t$, $y = 1 - \cos t$, $z = 4\cos\frac{t}{2}$



(2) Vektorska jednadžba:

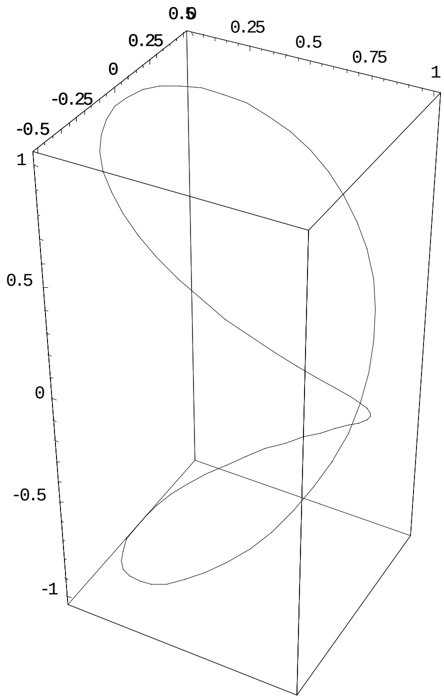
$$\vec{x} = \frac{1}{2}(1 + \cos t)\,\vec{i} + \frac{1}{2}\sin t\,\vec{j} + \frac{1}{2}\sin\frac{t}{2}\,\vec{k} = \left(\frac{1}{2}(1 + \cos t), \frac{1}{2}\sin t, \frac{1}{2}\sin\frac{t}{2}\right), \quad t \in [0, 4\pi]$$

Parametarske jednadžbe: $x = \frac{1}{2}(1 + \cos t)$, $y = \frac{1}{2}\sin t$, $z = \frac{1}{2}\sin\frac{t}{2}$



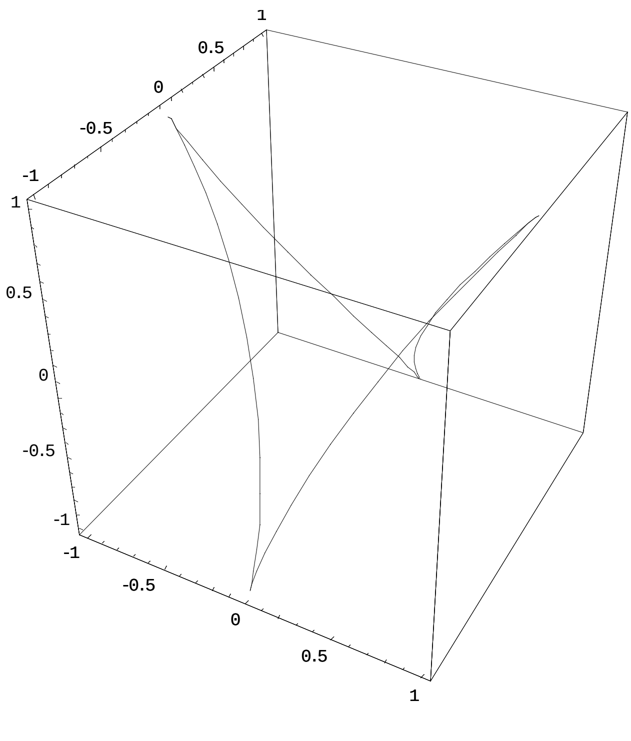
(3) Vektorska jednadžba: $\vec{x} = \cos^2 t\,\vec{i} + \frac{1}{2}\sin 2t\,\vec{j} + \sin t\,\vec{k} = \left(\cos^2 t, \frac{1}{2}\sin 2t, \sin t\right), \quad t \in [0, 2\pi]$

Parametarske jednadžbe: $x = \cos^2 t\,, \quad y = \frac{1}{2}\sin 2t\,, \quad z = \sin t$



(4) Vektorska jednadžba: $\vec{x} = \cos^3 t\,\vec{i} + \sin^3 t\,\vec{j} + \cos 2t\,\vec{k} = \left(\cos^3 t, \sin^3 t, \cos 2t\right), \quad t \in [0, 2\pi]$

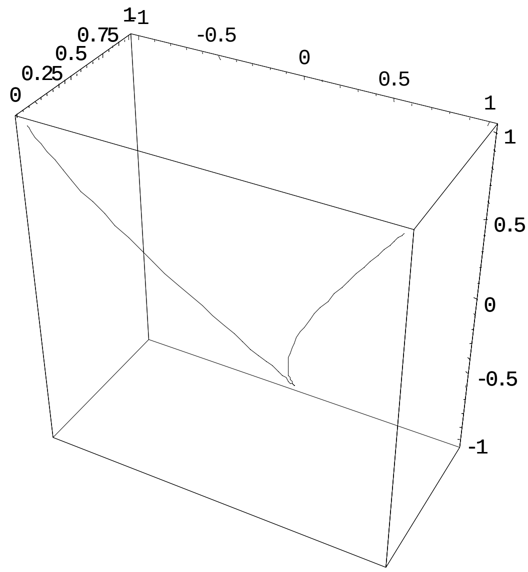
Parametarske jednadžbe: $x = \cos^3 t\,, \quad y = \sin^3 t\,, \quad z = \cos 2t$



Da bismo bolje shvatili kako nastaje graf zadane krivulje, pogledajmo kako izgleda graf te krivulje za $t \in [0, \pi]$, odnosno za $t \in [\pi, 2\pi]$

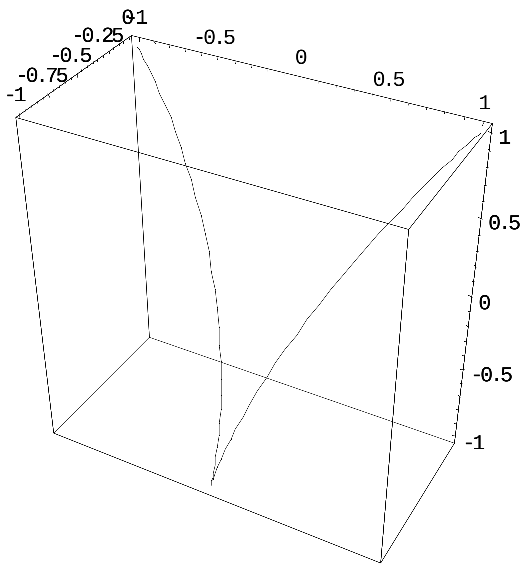
(4.1) Vektorska jednadžba:

$$\vec{x} = \cos^3 t\,\vec{i} + \sin^3 t\,\vec{j} + \cos 2t\,\vec{k} = \left(\cos^3 t, \sin^3 t, \cos 2t\right), \quad t \in [0, \pi]$$



(4.2) Vektorska jednadžba:

$$\vec{x} = \cos^3 t\,\vec{i} + \sin^3 t\,\vec{j} + \cos 2t\,\vec{k} = \left(\cos^3 t, \sin^3 t, \cos 2t\right), \quad t \in [\pi, 2\pi]$$



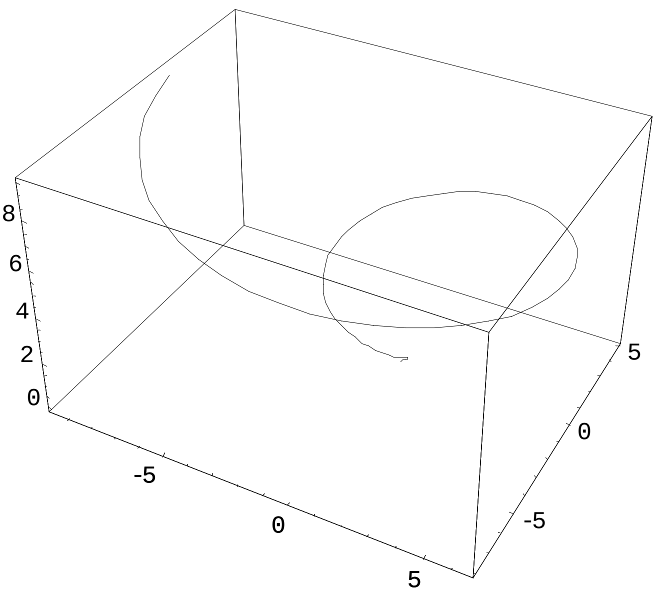
- Pogledajmo sada graf funkcije $\vec{x}: \mathrm{I} \to \mathbb{R}^3$ zadane sa:

$$\vec{x} = t\cos t\,\vec{i} + -t\sin t\,\vec{j} + t\,\vec{k} = \left(t\cos t, -t\sin t, t\right)$$

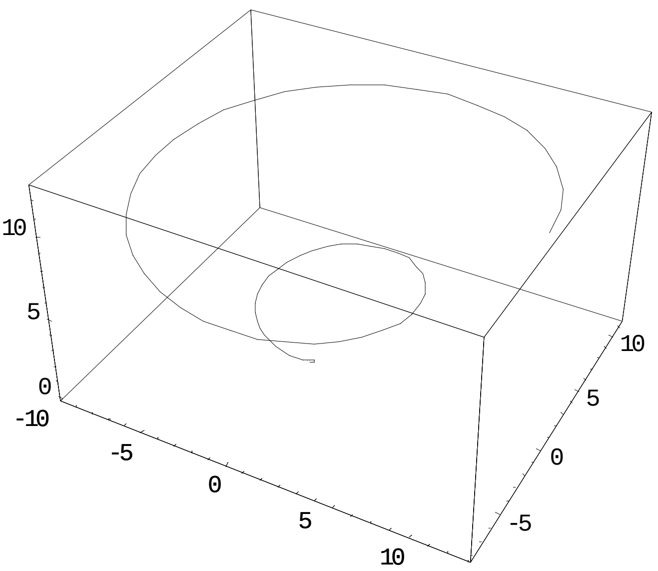
za neke proizvoljne intervale (segmente) I.

Jasno, parametarske jednadžbe su: $x = t\cos t$, $y = -t\sin t$, $z = t$.

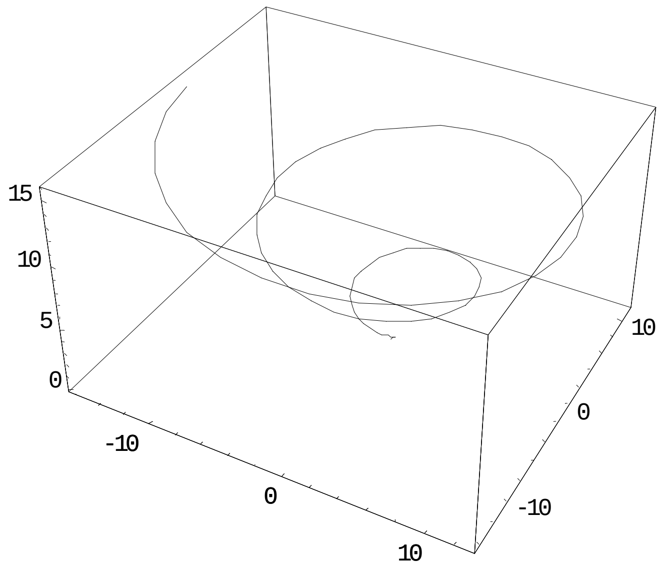
(5.1) Vektorska jednadžba: $\vec{x} = t\cos t\,\vec{i} + -t\sin t\,\vec{j} + t\,\vec{k} = \left(t\cos t, -t\sin t, t\right)$, $\mathrm{I} = \left[0, 3\pi\right]$



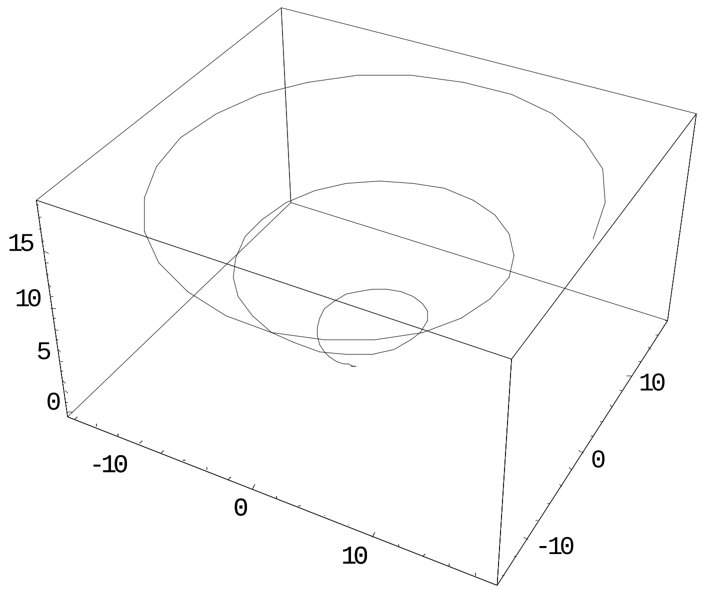
(5.2) Vektorska jednadžba: $\vec{x} = t\cos t\,\vec{i} + -t\sin t\,\vec{j} + t\,\vec{k} = \left(t\cos t, -t\sin t, t\right)$, $\mathrm{I} = \left[0, 4\pi\right]$



(5.3) Vektorska jednadžba: $\vec{x} = t\cos t\,\vec{i} + -t\sin t\,\vec{j} + t\,\vec{k} = (t\cos t, -t\sin t, t)$, $\mathrm{I} = [0, 5\pi]$

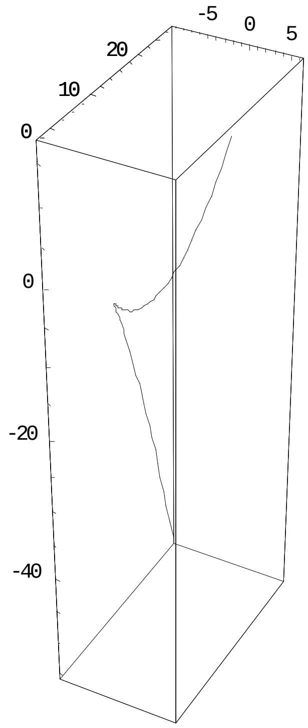


(5.4) Vektorska jednadžba: $\vec{x} = t\cos t\,\vec{i} + -t\sin t\,\vec{j} + t\,\vec{k} = (t\cos t, -t\sin t, t)$, $\mathrm{I} = [0, 6\pi]$



(6) Vektorska jednadžba: $\vec{x} = 3t\,\vec{i} + 3t^2\,\vec{j} + 2t^3\,\vec{k} = \left(3t, 3t^2, 2t^3\right), \quad t \in \mathrm{I} = [-3, 2]$

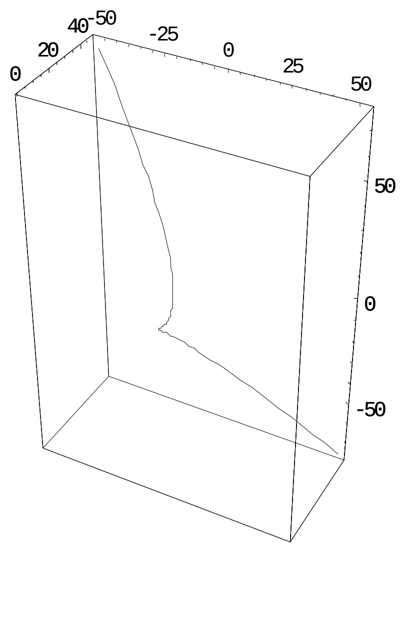
Parametarske jednadžbe: $x = 3t, \quad y = 3t^2, \quad z = 2t^3$



(7) Vektorska jednadžba:

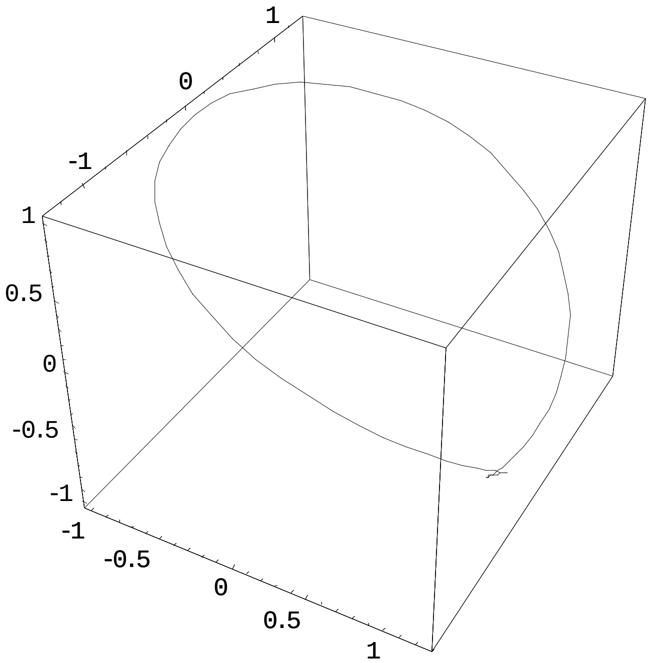
$$\vec{x} = \left(3t - t^3\right)\vec{i} + 3t^2\,\vec{j} + \left(3t + t^3\right)\vec{k} = \left(3t - t^3, 3t^2, 3t + t^3\right), \quad t \in \mathrm{I} = [-4, 4]$$

Parametarske jednadžbe: $x = 3t - t^3, \quad y = 3t^2, \quad z = 3t + t^3$



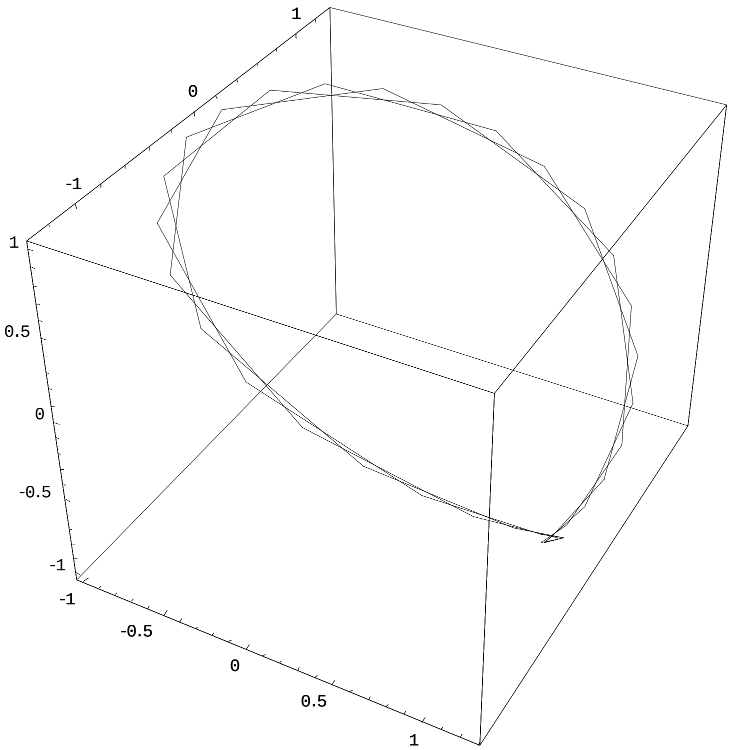
(8.1) Vektorska jednadžba:

$$\vec{x}=\left(\cos t+\sin^2 t\right)\vec{i}+\sin t\cdot\left(1-\cos t\right)\,\vec{j}-\cos t\,\vec{k}=\left(\cos t+\sin^2 t,\sin t\cdot\left(1-\cos t\right),-\cos t\right),\qquad t\in\left[0,2\pi\right]$$



(8.2) Vektorska jednadžba:

$$\vec{x}=\left(\cos t+\sin^2 t\right)\vec{i}+\sin t\cdot\left(1-\cos t\right)\,\vec{j}-\cos t\,\vec{k}=\left(\cos t+\sin^2 t,\sin t\cdot\left(1-\cos t\right),-\cos t\right),\quad t\in\left[0,12\pi\right]$$



Pripadne parametarske jednadžbe su:

$$x=\cos t+\sin^2 t,\quad y=\sin t\cdot\left(1-\cos t\right),\quad z=-\cos t\,.$$