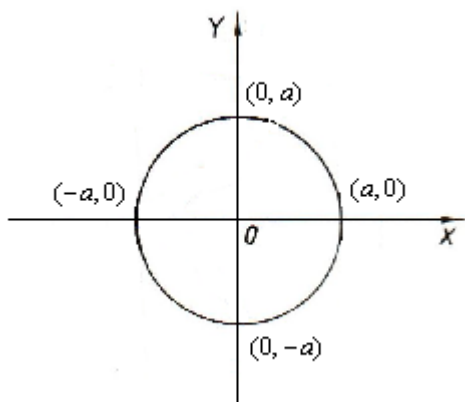


# Dodatak - Primjeri krivulja u ravni

## 1. Krivulje drugog reda

**Kružnica**

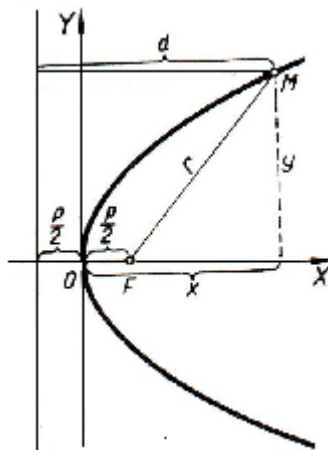


Jednadžba:  $x^2 + y^2 = a^2$

Parametarske jednadžbe:

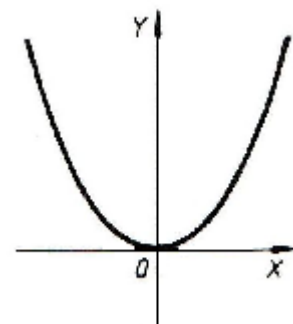
$$\begin{cases} x = a \cos t \\ y = a \sin t \end{cases}$$

**Parabola**



Jednadžba:  $y^2 = 2px$

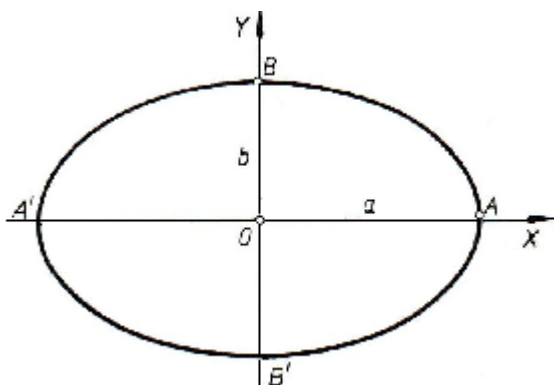
$$\begin{cases} x = t^2 \\ y = \sqrt{2p} \cdot t \end{cases}$$



Jednadžba:  $y = ax^2$

$$\begin{cases} x = t \\ y = at^2 \end{cases}$$

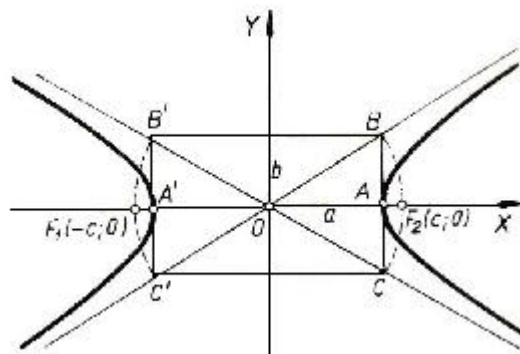
**Elipsa**



Jednadžba:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Parametarske jednadžbe:  $\begin{cases} x = a \cos t \\ y = b \sin t \end{cases}$

**Hiperbola**

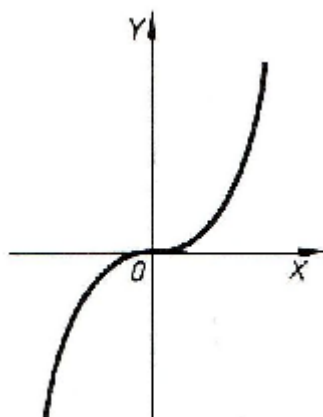


Jednadžba:  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Parametarske jednadžbe:  $\begin{cases} x = a \operatorname{ch} t \\ y = b \operatorname{sh} t \end{cases}$  (za desnu granu)

## 2. Krivulje trećeg reda

### Kubna parabola

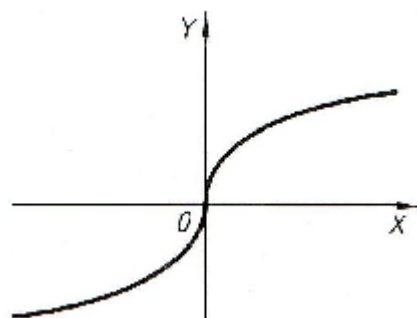


Jednadžba:  $y = ax^3$

Parametarske jednadžbe:  $\begin{cases} x = t \\ y = at^3 \end{cases}$

U ishodištu: točka infleksije

### Kubna parabola

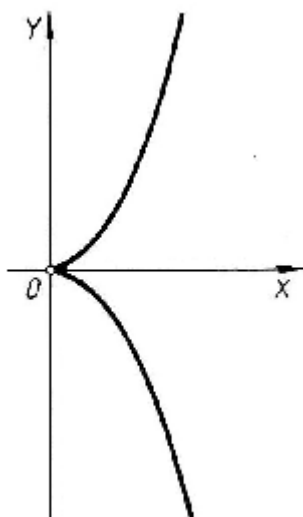


$y = ax^{1/3}$

$\begin{cases} x = t^3 \\ y = at \end{cases}$

točka infleksije

### Semikubna parabola

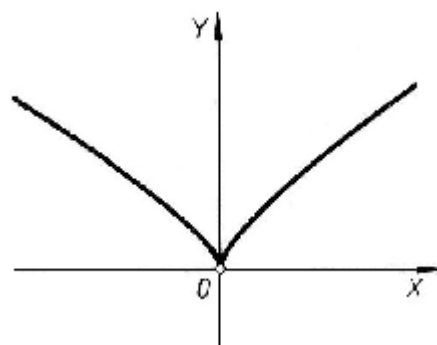


Jednadžba:  $y = ax^{3/2}$

Parametarske jednadžbe:  $\begin{cases} x = t^2 \\ y = at^3 \end{cases}$

U ishodištu: šiljak (singularna točka)

### Neilova parabola

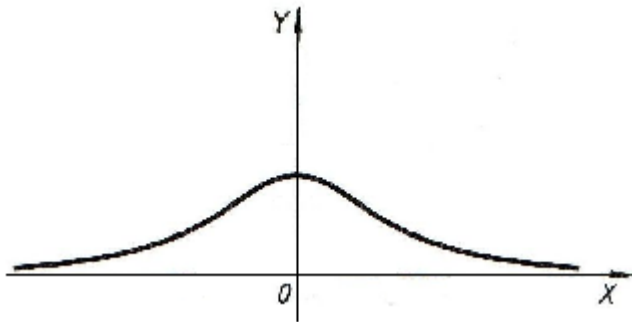


$y = ax^{2/3}$

$\begin{cases} x = t^3 \\ y = at^2 \end{cases}$

šiljak (singularna točka)

### "Versiera" Marije Agnesi

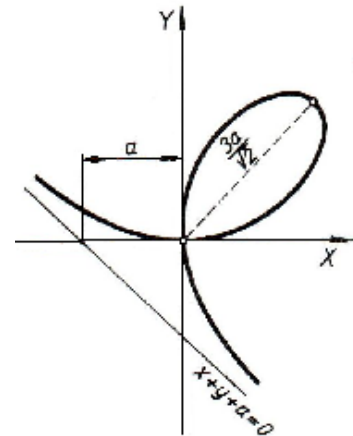


Jednadžba:  $y = \frac{a^3}{a^2 + x^2}$

Parametarske jednadžbe:

Točke infleksije:  $\left(\pm \frac{a}{\sqrt{3}}, \frac{3a}{4}\right)$

### Descartesov list

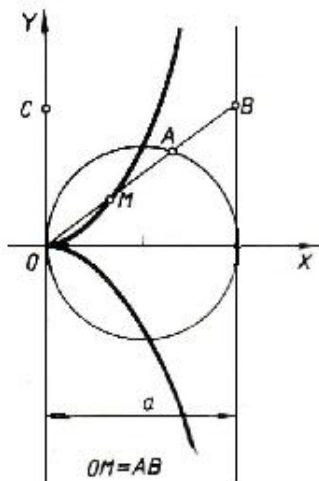


$x^3 + y^3 = 3axy$

$$\begin{cases} x = \frac{3at}{1+t^3} \\ y = \frac{3at^2}{1+t^3} \end{cases}$$

U ishodištu: dvostruka (singularna) točka

### Dioklova cisoida

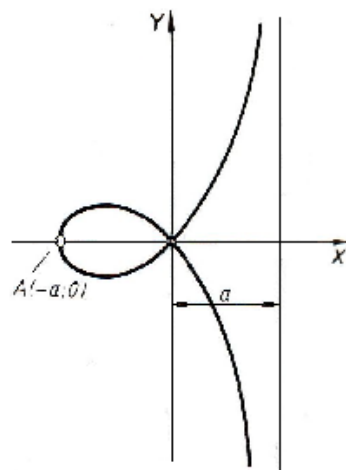


Jednadžba:  $y^2 = \frac{x^3}{a - x}$

Parametarske jednadžbe: 
$$\begin{cases} x = \frac{at^2}{t^2 + 1} \\ y = \frac{at^3}{t^2 + 1} \end{cases}$$

U ishodištu: šiljak (singularna točka)

### Strofoida



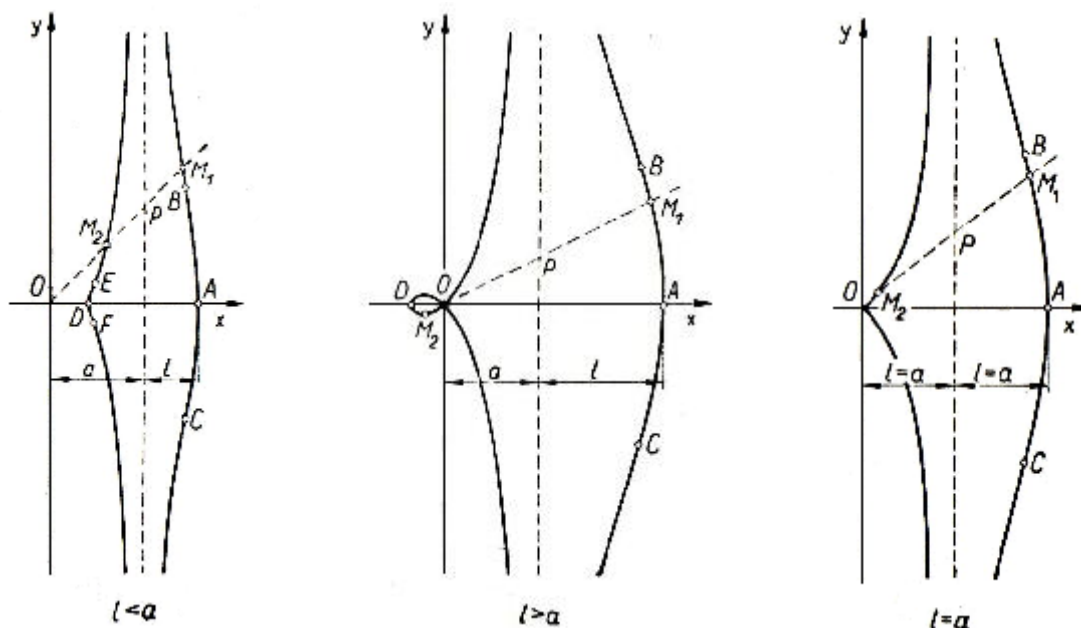
$y^2 = \frac{x^2 \cdot (a + x)}{a - x}$

$$\begin{cases} x = \frac{a(t^2 - 1)}{t^2 + 1} \\ y = \frac{at(t^2 - 1)}{t^2 + 1} \end{cases}$$

dvostruka (singularna) točka.

### 3. Krivulje četvrtog reda

#### Nikomedova konhoida



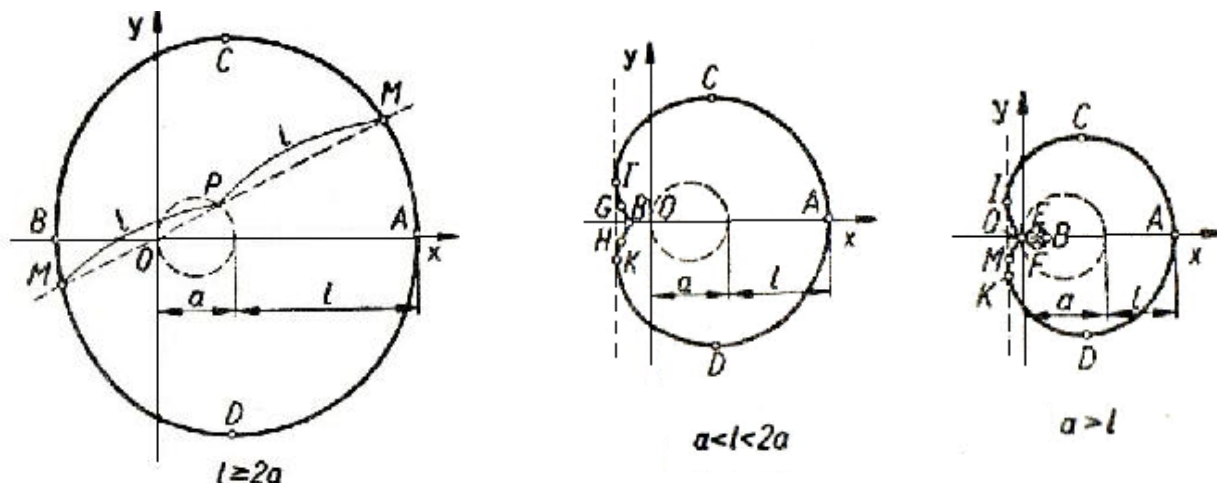
Implicitna jednačba:  $(x-a)^2 \cdot (x^2 + y^2) - l^2 x^2 = 0$

Parametarske jednačbe: 
$$\begin{cases} x = a + l \cos \varphi \\ y = a \operatorname{tg} \varphi + l \sin \varphi \end{cases}$$

Polarna jednačba:  $\rho = \frac{a}{\cos \varphi} \pm l$

U ishodištu (singularne točke):  
 izolirana točka ako je  $l < a$ ,  
 čvorna točka ako je  $l > a$ ,  
 šiljak ako je  $l = a$ .

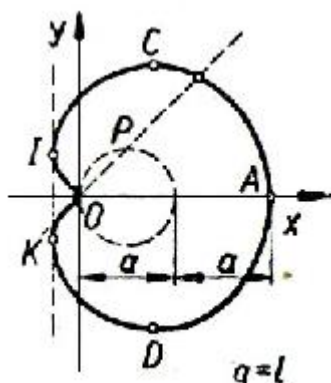
#### Pascalov puž (konhoida kružnice)



Implicitna jednačba:  $(x^2 + y^2 - ax)^2 - l^2 (x^2 + y^2) = 0$ . Polarna jednačba:  $\rho = a \cos \varphi + l$

Parametarske jednačbe:  $x = a \cos^2 \varphi + l \cos \varphi$ ,  $y = a \cos \varphi \sin \varphi + l \sin \varphi$

**Kardioida** (poseban slučaj Pascalovog puža za  $a=l$ )

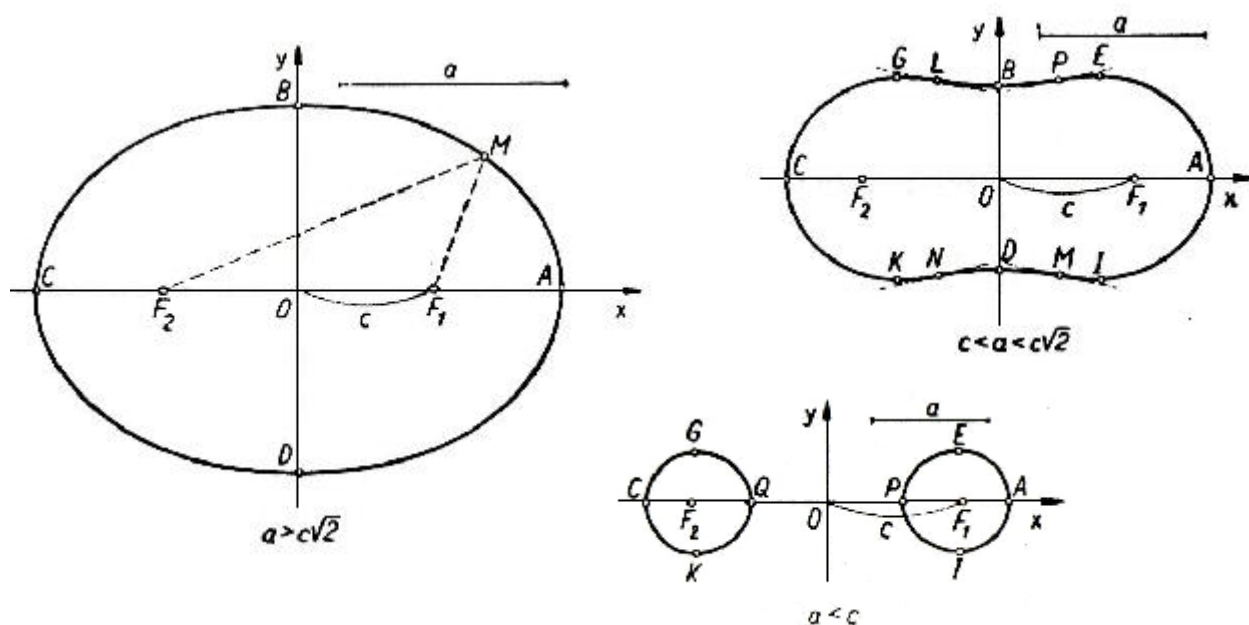


Implicitna jednačba:  $(x^2 + y^2)^2 - 2ax(x^2 + y^2) - a^2y^2 = 0$

Parametarske jednačbe: 
$$\begin{cases} x = a \cos \varphi (1 + \cos \varphi) \\ y = a \sin \varphi (1 + \cos \varphi) \end{cases}$$

Polarna jednačba:  $\rho = a(1 + \cos \varphi)$

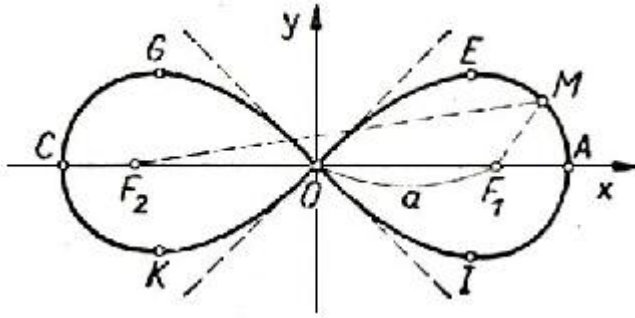
### Cassinijevi ovali




Implicitna jednačba:  $(x^2 + y^2)^2 - 2c^2(x^2 - y^2) - a^4 + c^4 = 0$

Polarna jednačba:  $\rho^2 = c^2 \cos 2\varphi \pm \sqrt{c^4 \cos^2 2\varphi + (a^4 - c^4)}$ .

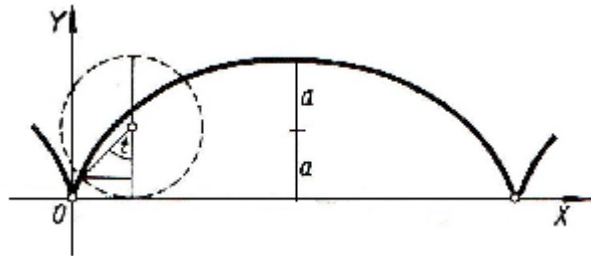
**Lemniskata** (poseban slučaj Cassinijevih ovala za  $a = c$ )



Implicitna jednačina:  $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) = 0$ . Polarna jednačina:  $\rho = a\sqrt{2\cos 2\varphi}$ .

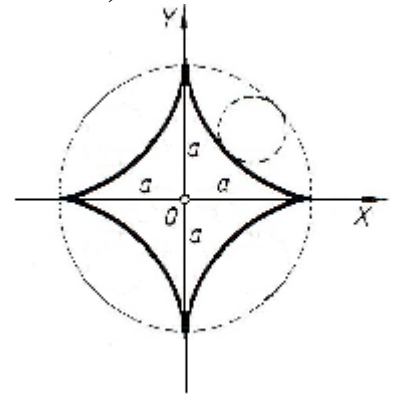
 Navedimo još neke krivulje:

**Cikloida**



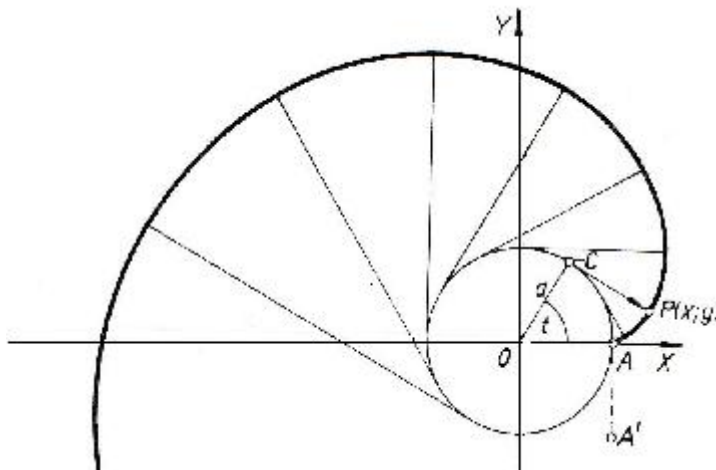
Parametarske jednačine:  $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$

**Astroida (hipocikloida)**



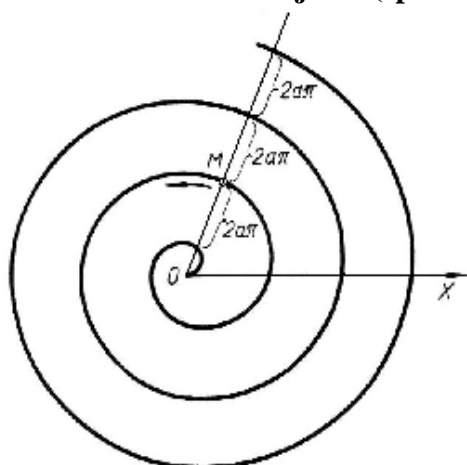
$\begin{cases} x = a \cos^3 t \\ y = a \sin^3 t \end{cases}$   
 Implicitna jednačina:  $x^{2/3} + y^{2/3} = a^{2/3}$

**Evolventa kružnice**



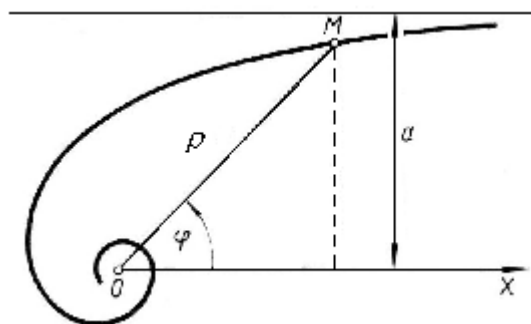
Parametarske jednačine:  $x = a(\cos t + t \sin t)$ ,  $y = a(\sin t - t \cos t)$

### Arhimedova zavojnica (spirala)



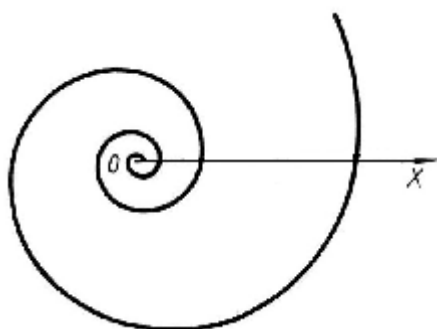
Polarna jednadžba:  $\rho = a \cdot \varphi, (\varphi \geq 0)$ .

### Hiperbolna zavojnica



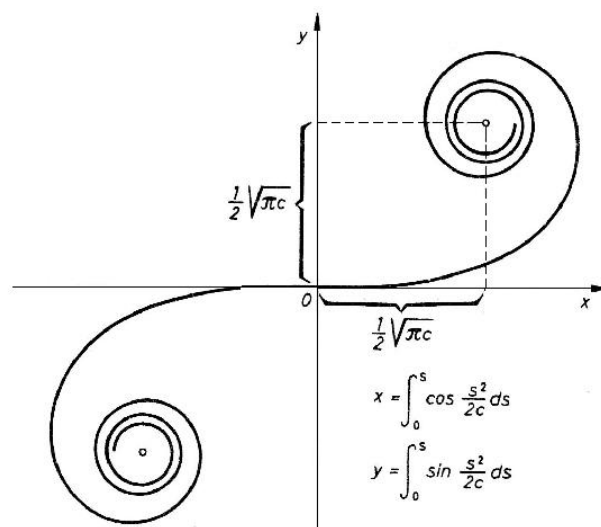
$$\rho = \frac{a}{\varphi}, (\varphi > 0).$$

### Logaritamska zavojnica (spirala)

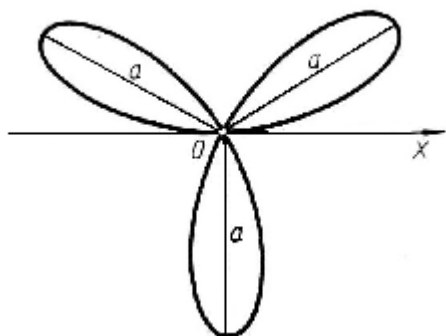


Polarna jednadžba:  $\rho = e^{a \cdot \varphi}$ .

### Klotoida

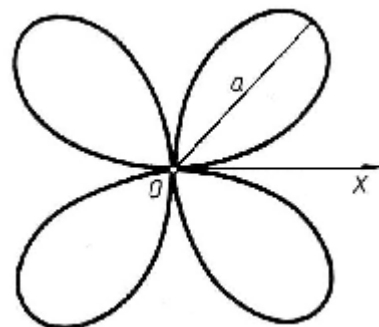


### Ruža sa tri latice



Polarna jednadžba:  $\rho = a \sin 3\varphi, (\varphi \geq 0)$ .

### Ruža sa četiri latice



$$\rho = a \sin 2\varphi$$