Recurrence and transience property for stable-like Markov chains

(Talk)

Nikola Sandrić
Faculty of Civil Engineering, University of Zagreb, Croatia
nsandric@grad.hr

Let $\{S_n\}_{n\geq 0}$ be a random walk on $\mathbb{R}^d$, $d \geq 1$. The random walk $\{S_n\}_{n\geq 0}$ is said to be recurrent if

$$\mathbb{P}\left(\liminf_{n\longrightarrow\infty} |S_n| = 0\right) = 1,$$

and transient if

$$\mathbb{P}\left(\lim_{n\longrightarrow\infty} |S_n| = \infty\right) = 1.$$

It is well known that every random walk is either recurrent or transient. In the class of α-stable random walks on $\mathbb{R}$, a symmetric α-stable random walk is recurrent if and only if $\alpha \geq 1$. We generalize one-dimensional symmetric α-stable random walk in the way that the index of stability of jump distribution depends on the current position and we study the recurrence and transience property of the generalization. In other words, we consider the recurrence and transience problem for a temporally homogeneous Markov chain on the real line with transition function $p(x, dy) = f_x(y - x)dy$, where the density functions $f_x(y)$, for large $|y|$, have a power-law decay with exponent $\alpha(x) + 1$, where $\alpha(x) \in (0, 2)$.

Under a uniformity condition on the densities $f_x(y)$ and some mild technical conditions, we prove that when $\liminf_{|x|\longrightarrow\infty} \alpha(x) > 1$, the chain is recurrent, while when $\limsup_{|x|\longrightarrow\infty} \alpha(x) < 1$, the chain is transient.

Furthermore, if $f_x(y)$ are densities of symmetric distributions such that the function $x \longmapsto f_x$ is periodic and the set $\{x : \alpha(x) = \alpha_0 := \inf_{x\in\mathbb{R}} \alpha(x)\}$ has positive Lebesgue measure, then, under some mild technical conditions on the densities $f_x(y)$, the chain is recurrent if and only if $\alpha_0 \geq 1$.

Finally, if $f_x(y)$ is the density of a symmetric α-stable distribution for negative x and the density of a symmetric β-stable distribution for non-negative x, where $\alpha, \beta \in (0, 2)$, then the chain is recurrent if and only if $\alpha + \beta \geq 2$.

As a special case of these results we give a new proof for the recurrence and transience property of a symmetric α-stable random walk on $\mathbb{R}$ with the index of stability $\alpha \in (0, 2)$.

MSC2010: 60G52, 60J05, 60J25.

Keywords: characteristics of semimartingale, Feller process, Foster-Lyapunov drift criterion, Harris recurrence, Markov chain, Markov process, petite set, recurrence, stable distribution, stable-like process, transience.

Section: Probability and Statistics.

.