Strong and weak solutions of quasilinear elliptic equations with strong dependence on the gradient

(Talk)

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We study the existence and nonexistence of positive, spherically symmetric solutions of the next quasilinear elliptic equation with an arbitrary positive growth rate e_0 on the gradient on the right-hand side:

$$\begin{cases} -\Delta_p u = \tilde{g_0}|x|^m + \tilde{f_0}|\nabla u|^{e_0} \quad \text{in } B \setminus \{0\}, \\ \quad u = 0 \text{ on } \partial B, \\ u(x) \text{ spherically symmetric and decreasing.} \end{cases} \tag{1}$$

Here B is an open ball of radius R centered at the origin in $\mathbf{R^N}$, $1 < p < \infty$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is p-Laplacian. The Lebesgue measure (volume) of B in $\mathbf{R^N}$ is denoted by $|B|$, and the volume of the unit ball is denoted by C_N. The dual exponent of $p > 1$ is defined by $p' = \frac{p}{p-1}$. We assume that $\tilde{g_0}$, $\tilde{f_0}$ and e_0 are positive real numbers. We show that $e_0 = p - 1$ is the critical exponent: for $e_0 < p - 1$ there exists a strong solution for any choice of the coefficients which is a known result, while for $e_0 > p - 1$ we have existence-nonexistence splitting of the coefficients $\tilde{f_0}$ and $\tilde{g_0}$. The elliptic problem is studied by relating it to the corresponding singular ODE of the first order.We give a sufficient conditions for a strong radial solution to be the weak solution.We also examined when ω-solutions of (1) are weak solutions. We found conditions under which the strong solutions are the weak solutions in the critical case of $e_0 = p - 1$.

References

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