Some new kinds of attractivity for nonautonomous differential systems

Talk

Mervan Pašić
University of Zagreb
mervan.pasic@fer.hr

For a nonautonomous linear system (NS): $\mathbf{x}' = A(t)\mathbf{x}$, $t \in (0, t_0]$, $\mathbf{x}(t_0) = \mathbf{x}_0 \in \mathbb{R}^2$, the zero solution $(0,0)$ is attractive as $t \to 0$ if $\|\mathbf{x}(t)\| \to 0$ as $t \to 0$ for all solution $\mathbf{x}$. Moreover, if the length of the corresponding solution curve $\Gamma_{\mathbf{x}} \subseteq \mathbb{R}^2$ associated to $\mathbf{x}$ is finite (resp. infinite) for all solution $\mathbf{x}$, then the zero solution is said to be rectifiable (resp. nonrectifiable) attractive as $t \to 0$. Furthermore, if there is a real number $s \in (1,2)$ such that $\dim_M(\Gamma_{\mathbf{x}}) = s$ and $0 < M_*^s(\Gamma_{\mathbf{x}}) \le M^{*s}(\Gamma_{\mathbf{x}}) < \infty$, then the zero solution is said to be fractal attractive as $t \to 0$. Here $\dim_M(\Gamma_{\mathbf{x}})$, $M_*^s(\Gamma_{\mathbf{x}})$ and $M^{*s}(\Gamma_{\mathbf{x}})$ denote respectively the box-counting (Minkowski-Bouligand) dimension, lower and upper Minkowski contents of $\Gamma_{\mathbf{x}}$. These new kinds of attractivity for the system (NS) is studied in the dependence on asymptotic behaviour of the eigenvalues of matrix $A(t)$, which is a consequence of the presumed singularity of $A(t)$ near $t = 0$. It is based on some papers recently written by Mervan Pašić, Yuki Naito and Satoshi Tanaka.

MSC2010: 26B15, 28A75, 34A30, 34D05, 51M25, 53A04.

Keywords: nonautonomous system, attractive zero solution, asymptotic behaviour, fractal curves, fractal dimension, Minkowski content.

Section: 12.

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