Linear singular differntial equations in Banach space and nonrectifiable attractivity in two-dimensional linear differential systems

Talk

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(joint work with prof. dr. sc. Mervan Pašić)

We study the asymptotic behaviour near $t = 0$ of all solutions $\mathbf{x} \in C^1((0, t_0]; \mathbb{X})$ of linear nonautonomous differential equation

$$\mathbf{x}' = A(t)\mathbf{x},\ t \in (0, t_0] \tag{1}$$

where $\mathbb{X}$ is an arbitrary Banach space and $A\colon (0, t_0] \to L(\mathbb{X})$ is an operator-valued function which may be singular at $t = 0$. In terms of some smyptotic behaviour of the operator norm $\|A(t)\|$ near $t = 0$, the kind of singularity (resp. regularity) of equation (1) is characterized: for every $\mathbf{x}_0 \in \mathbb{X}$ and solution $\mathbf{x}$ of (1) such that $\mathbf{x}(t_0) = x_0$, we have $\|\mathbf{x}(t)\|_{\mathbb{X}} \to 0$ as $t \to 0$ and $\|x'\|_{\mathbb{X}} \notin L^1((0, t_0])$ (resp. $\|x'\|_{\mathbb{X}} \in L^1((0, t_0])$). Next, when $\mathbb{X} = \mathbb{R}^2$ and equation (1) is a two-dimensional linear integrable differntial system, our previous result allows us to characterize the so-called nonrectifiable (resp. rectifiable) attractivity of zero the zero solution to the equation (1), that is $\|\mathbf{x}(t)\|_{\mathbb{R}^2} \to 0$ as $t \to 0$, the solution's curve $\Gamma_{\mathbf{x}}$ is a Jordan curve in $\mathbb{R}^2$ and length$(\Gamma_{\mathbf{x}}) = \infty$ (resp. length$(\Gamma_{\mathbf{x}}) < \infty$).

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