COMPUTATION OF CONSTANTS IN MULTIPARAMETRIC ALGEBRAS OF NONCOMMUTATIVE POLYNOMIALS

(Talk)

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Let $\mathbb{N}_0 = \{0, 1, \ldots\}$ be the set of nonnegative integers and let $\mathcal{N} = \{i_1, \ldots, i_N\}$ be a fixed subset of $\mathbb{N}_0$. Then we denote by $\mathcal{B} = \mathcal{B}_{\mathcal{N}} = \mathbb{C}\langle e_{i_1}, \ldots, e_{i_N}\rangle$ the free unital associative $\mathbb{C}$-algebra with N generators $\{e_i\}_{i\in\mathcal{N}}$, each of degree one. We can think of $\mathcal{B}$ as an algebra of noncommutative polynomials in N noncommuting variables $e_{i_1}, \ldots, e_{i_N}$.
We equip $\mathcal{B}$ with a multiparametric q_{ij}-differential structure given by N linear operators $\partial_i \colon \mathcal{B} \to \mathcal{B}$, $i \in \mathcal{N}$ that act as twisted derivations on $\mathcal{B}$:
$\partial_i(1) = 0,\ \partial_i(e_j) = \delta_{ij},\ \partial_i(e_j x) = \delta_{ij} x + q_{ij} e_j \partial_i(x)$ for all $x \in \mathcal{B}$, $i, j \in \mathcal{N}$
(q_{ij} are complex numbers).
The algebra $\mathcal{B}$ is naturally graded by total degree $\mathcal{B} = \bigoplus_{n\geq 0} \mathcal{B}^n$, where $\mathcal{B}^0 = \mathbb{C}$ and $\mathcal{B}^n$ consists of all homogeneous noncommuting polynomials of total degree n in variables $e_{i_1}, \ldots, e_{i_N}$. More generally we also have a finer decomposition of $\mathcal{B}$ into multigraded components (= weight subspaces)

$$\mathcal{B} = \bigoplus_{n\geq 0,\, l_1\leq\cdots\leq l_n,\, l_j\in\mathcal{N}} \mathcal{B}_{l_1\ldots l_n},$$

where each weight subspace $\mathcal{B}_Q = \mathcal{B}_{l_1\ldots l_n}$, corresponds to a multiset $Q = (l_1 \ldots l_n)$, is given by

$$\mathcal{B}_Q = span_{\mathbb{C}}\left\{ e_{j_1\ldots j_n} := e_{j_1}\cdots e_{j_n} \mid j_1 \ldots j_n \in \widehat{Q} \right\}.$$

Here $\widehat{Q} = S_n Q = \{\sigma(l_1 \ldots l_n) \mid \sigma \in S_n\}$ denotes the set of all rearrangements of the sequence $l_1, \ldots, l_n$ (i.e $\widehat{Q}$ is the set of all distinct permutations of the multiset Q). Thus $\dim \mathcal{B}_Q = \left|\widehat{Q}\right|$.
Of particular interest in algebra $\mathcal{B}$ are elements called *constants* which satisfy $\partial_i C = 0$ for every $i \in \mathcal{N}$. Let $\mathcal{C}$ denotes the space of all constants in algebra $\mathcal{B}$ and similarly let $\mathcal{C}_Q$ denotes the space of all constants in $\mathcal{B}_Q$. Then the main problem of describing the space $\mathcal{C}$ can be reduced to describing the space $\mathcal{C}_Q$. Here we shall give the explicit formulas for nontrivial (basic) constants in $\mathcal{B}_Q$ up to total degree equal to four.

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