# Intrinsic Geometry of Cyclic Polygons via "New" Brahmagupta Formula

(Talk )

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Finding explicit equations for the area or circumradius of polygons inscribed in a circle in terms of side lengths is a classical subject (cf.[1]). For triangle / cyclic quadrilaterals we have famous Heron / Brahmagupta formulae. In 1994. D.P. Robbins found a minimal area equations for cyclic pentagons/hexagons by a method of undetermined coefficients(cf.[3]). This method could hardly be used for heptagons due to computational complexity (143307 equations). In [4], by using covariants of binary quintics, a concise minimal heptagon/octagon area equation was obtained as a fraction of two resultants which in expanded form has almost one milion terms. It is not clear if this approach could be effectively used for cyclic polygons with nine or more sides . In [6], by using Wiener-Hopf factorization approach , we have obtained a very explicit minimal heptagon/octagon circumradius equation in Pellian form with coefficients up to four digits. A nonminimal area equation is also obtainable by this method. Both methods are somehow external. But,based on our new intermediate Brahmagupta formula, we have succeded also in finding an intrinsic proof of the Robbins formula for the area (and also for circumradius and area times circumradius) of cyclic hexagon based on an intricate direct elimination of diagonals (the case of pentagon was much easier cf. [5]).We also get a simple(st) system of equations for the area and area times circumradius of cyclic heptagons /octagons. It seems remarkable that our approach, with a help of Groebner basis techniques leads to minimal equations (for any concrete instances we have tested) ,what is not the case with iterated resultants approach.

## References

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MSC2010: 51Mxx,52Mxx.

Keywords: intrinsic proof, cyclic heptagon, cyclic octagon.

Section: 5.

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