Geometry of pentagonal quasigroups

(Talk)

Stipe Vidak

Department of Mathematics, University of Zagreb

`stipe.vidak@gmail.com`

Pentagonal quasigroups are idempotent medial quasigroups satisfying the additional identity of pentagonality, $(ab \cdot a)b \cdot a = b$. Basic example is $C(q) = (\mathbb{C}, *)$, where $*$ is binary operation on $\mathbb{C}$ defined by $a * b = (1 - q)a + qb$ for a, $b \in \mathbb{C}$, and q is a solution of the equation $q^4 - 3q^3 + 4q^2 - 2q + 1 = 0$. Using this example as motivation, some geometrical concepts, such as parallelogram, midpoint of a segment, regular pentagon and regular decagon, are defined in a general pentagonal quasigroup. These concepts and their mutual relations are studied and presented in $C(q)$ and in some finite pentagonal quasigroups of order 5 and 11. Using only algebraic identities which hold in pentagonal quasigroups many generalizations of theorems of the Euclidean plane can be proved in a general pentagonal quasigroup.

Section: Geometry and Differential Geometry.

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