# Asymptotic $K$-character of nilpotent orbits

(Talk)

Mladen Božičević
Geotehnički fakultet, Hallerova aleja 7, 42000 Varaždin
mbozicev@gfv.hr

Let $G$ be a complex reductive algebraic group and $K \subset G$ the fixed-point set of a regular involution on $G$. Denote by $\mathfrak{k} \subset \mathfrak{g}$ the corresponding Lie algebras, and by $\mathfrak{p}$ the Cartan complement of $\mathfrak{k}$ in $\mathfrak{g}$. Let $G_{\mathbb{R}}$ be a real form of $G$ and $\mathfrak{g}_{\mathbb{R}}$ the Lie algebra of $G_{\mathbb{R}}$. Write $K_{\mathbb{R}} = G_{\mathbb{R}} \cap K$ and $\mathfrak{k}_{\mathbb{R}} = \mathfrak{g}_{\mathbb{R}} \cap \mathfrak{k}$. Denote by $\mathcal{N}^*$ the cone of nilpotent elements in in the dual Lie algebra $\mathfrak{g}^*$, and consider a $G_{\mathbb{R}}$-orbit $\mathcal{V}$ in $\mathcal{N}^* \cap \mathfrak{g}_{\mathbb{R}}^*$ and a $K$-orbit $\mathcal{O}$ in $\mathcal{N}^* \cap \mathfrak{p}^*$ associated by the Kostant-Sekiguchi correspondence.

According to a conjecture of Vogan the asymptotic behaviour of multiplicities of $K_{\mathbb{R}}$-types in the ring of regular functions $R\left[\overline{\mathcal{O}}\right]$ can be described in terms of the Liouville measure $\beta_{\mathcal{V}}$ on $\mathcal{V}$. More precisely, we consider the generalised function $J_{\mathcal{V}}$ defined as restriction of the Fourier transform of $\beta_{\mathcal{V}}$ to $\mathfrak{k}_{\mathbb{R}}$. On the other hand, $R\left[\overline{\mathcal{O}}\right]$ is a trace class representation of $K_{\mathbb{R}}$, hence one can define the asymptotic $K_{\mathbb{R}}$-character of $\mathcal{O}$ as the limit

$$M_{\mathcal{O}}(X) = \lim_{t \to 0} t^d \operatorname{Tr}(R[\overline{\mathcal{O}}])(\exp tX), \quad d = \dim_{\mathbb{C}} \mathcal{O},\ X \in \mathfrak{k}_{\mathbb{R}}.$$

Vogan's conjecture states that $J_{\mathcal{V}} = cM_{\mathcal{O}}$ as generalised functions on $\mathfrak{k}_{\mathbb{R}}$. The conjecture was established by King for even nilpotent orbits and minimal nilpotent orbits, and by Vergne for complex groups. The aim of this talk is to provide additional evidence for Vogan's conjecture. Relying on the results of Schmid and Vilonen, we show that Vogan's conjecture is true for the real forms of $GL(n, \mathbb{C})$ and $SL(n, \mathbb{C})$. We also discuss Vogan's conjecture for certain classes of nilpotent orbits in other classical groups.

MSC2010: 22E46, 22E30.

Keywords: real Lie group, nilpotent coadjoint orbit.

Section: 7.