On refined Hardy-type inequalities with fractional integrals and fractional derivatives

(Talk)

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(joint work with Sajid Iqbal and Josip Pečarić)

Let $[a,b],(-\infty < a < b < \infty)$ be a finite interval on real axis $\mathbb{R}$. The Riemann-Liouville fractional integrals $I^{\alpha}_{a^+}f$ and $I^{\alpha}_{b^-}f$ of order $\alpha > 0$ are defined by

$$I^{\alpha}_{a^+}f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x f(y)(x-y)^{\alpha-1}dy, \quad (x > a)$$

and

$$I^{\alpha}_{b^-}f(x) = \frac{1}{\Gamma(\alpha)}\int_x^b f(y)(y-x)^{\alpha-1}dy, \quad (x < b)$$

respectively. Here $\Gamma(\alpha)$ is the Gamma function, i.e. $\Gamma(\alpha) = \int_0^\infty e^t\, t^{\alpha-1}dt$. G. H. Hardy proved that fractional integral operators are bounded in $L_p(a,b)$, $-\infty < a < b < \infty,\ 1 \le p \le \infty$, that is

$$\|I^{\alpha}_{a^+}f\|_p \le K\|f\|_p, \qquad \|I^{\alpha}_{b^-}f\|_p \le K\|f\|_p \tag{1}$$

where

$$K = \frac{(b-a)^\alpha}{\Gamma(\alpha+1)}.$$

Inequality (1) refers to as inequality of G. H. Hardy.

The aim of this talk is to present new more general Hardy-type inequalities for different kinds of fractional integrals and fractional derivatives like Riemann-Liouville fractional integrals, Caputo fractional derivative, fractional integral of a function with respect to an increasing function, Erdelyi-Kóber fractional integrals and Hadamard-type fractional integrals.

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