Superadditivity of some Jensen-type functionals

(Talk)

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(joint work with M. Krnić and J. Pečarić)

In 1996. S. S. Dragomir, J. Pečarić and L. E. Persson investigated discrete Jensen's functional

$$J(f, \mathbf{x}, \mathbf{p}) = \sum_{i=1}^{n} p_i f(x_i) - P_n f\left(\frac{\sum_{i=1}^{n} p_i x_i}{P_n}\right), \tag{1}$$

where $f : I \to \mathbb{R}$, $I \subset \mathbb{R}$, $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in I^n$ and $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ is a nonnegative $n-$tuple of real numbers with $P_n = \sum_{i=1}^{n} p_i > 0$. They proved that, under the assumption of convexity of f, functional (1) is superadditive and increasing on the set of all n-tuples $\mathbf{p}$ described above.
Motivated by their results, we prove the analogues ones for Jessen's functional, which generalizes (1) by means of positive linear functionals acting on linear class of real valued functions, as well as for McShane's functional, which is a multidimensional generalization of Jessen's functional. Consequently, we establish their lower and upper bounds expressed by the non weighted functionals of the same type. Such bounds enable us to obtain converses and refinements of the series of the classical inequalities, such are arithmetic-geometric inequality, Young's and Hölder's inequality in the difference and quotient form, as well as some other related inequalities.

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