Relative zeta functions of fractal sets in Euclidean spaces

(Talk)

Goran Radunović
University of Zagreb
goran.radunovic@fer.hr

(joint work with Michel L. Lapidus, University of California, Riverside and Darko Žubrinić, University of Zagreb)

We extend the definition of zeta functions introduced by M.L. Lapidus in Catania 2009 associated to bounded fractal sets to the case of unbounded fractal sets with respect to a set of finite Lebesgue measure. Let A be a possibly unbounded subset of $\mathbb{R}^N$ and Ω a subset of $\mathbb{R}^N$ of finite Lebesgue measure. We define the relative zeta function $\zeta_A(\cdot,\Omega)$ of A with respect to Ω as

$$\zeta_A(s,\Omega) = \int_{A_\delta \cap \Omega} d(x,A)^{s-N}\,dx.$$

Here δ is a fixed positive number, A_δ is the δ-neighbourhood of A, $d(x,A)$ is the Euclidean distance from x to A, s is the complex variable, and the integral is taken in the sense of Lebesgue. On the other hand, if we have a pair of sets A and Ω as above, we can define the upper d-dimensional relative Minkowski content of A with respect to Ω:

$$\mathcal{M}^{*d}(A,\Omega) = \limsup_{\delta\to 0} \frac{|A_\delta \cap \Omega|}{\delta^{N-d}},$$

and define the upper relative box dimension $\overline{\dim}_B(A,\Omega)$ as the infimum of all d for which the upper relative Minkowski content is zero. We show that $\zeta_A(s,\Omega)$ is analytic on the right half-plane $\mathrm{Re}(s) \geq \overline{\dim}_B(A,\Omega)$. Moreover, this bound is optimal. We will illustrate the proof and show a few examples.

Reference:

M.L. Lapidus, G. Radunović, D. Žubrinić, Zeta functions of fractal sets in Euclidean spaces, in preparation.

MSC2010: 28A12, 30D30, 11M41.

Keywords: fractal set, box dimension, reduced complex dimension, Minkowski content, singular integral, harmonic function.

Section: 8.

.