Generalizations of Ostrowski inequality involving real Borel measures

(Talk)

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Let $M[a,b]$ be the Banach space of all real Borel measures on $[a,b]$ with the total variation norm. For $\mu \in M[a,b]$ define function $\check{\mu}_n : [a,b] \to \mathbf{R}$, $n \geq 1$, by

$$\check{\mu}_n(t) = \frac{1}{(n-1)!} \int_{[a,t]} (t-s)^{n-1} d\mu(s).$$

A sequence of functions $P_n : [a,b] \to \mathbb{R}$, $n \geq 1$, is called a **μ-harmonic sequence of functions** on $[a,b]$ if

$$P_1(t) = c + \check{\mu}_1(t), \ a \leq t \leq b,$$

for some $c \in \mathbb{R}$, and

$$P_{n+1}(t) = P_{n+1}(a) + \int_a^t P_n(s) ds, \ a \leq t \leq b, \ n \geq 1.$$

Define function $K_n : [a,b] \times [a,b] \to \mathbb{R}$, $n \geq 1$, by

$$K_n(x,t) = \begin{cases} P_n(b-x+t), & a \leq t \leq x \\ P_n(a-x+t), & x < t \leq b \end{cases}$$

for $a \leq x < b$, while for $x = b$

$$K_n(b,t) = \begin{cases} P_n(t), & a \leq t < b \\ P_n(a), & t = b \end{cases} .$$

The following theorem is the key result in this talk.

Theorem. For $\mu \in M[a,b]$ let $(P_n, \ n \geq 1)$ be a μ-harmonic sequence of functions on $[a,b]$ and $f : [a,b] \to \mathbb{R}$ such that $f^{(n-1)}$ is a continuous function of bounded variation for some $n \geq 1$. Then we have

$$\int_{[a,b]} f_x(t) d\mu(t) - \mu(\{a\}) f(x) + S_n(x) = R_n(x), \tag{1}$$

for every $x \in [a,b]$, where

$$f_x(t) = \begin{cases} f(x-a+t), & a \leq t \leq a+b-x \\ f(x-b+t), & a+b-x < t \leq b \end{cases} ,$$

$$S_n(x) = \sum_{k=1}^{n-1}(-1)^k P_k(a+b-x)\left[f^{(k-1)}(b) - f^{(k-1)}(a)\right] + \sum_{k=1}^{n}(-1)^k f^{(k-1)}(x)\left[P_k(b) - P_k(a)\right]$$

and

$$R_n(x) = (-1)^n \int_{[a,b]} \left[K_n(x,t) - K_n(x,a)\right] df^{(n-1)}(t).$$

Using Euler identity (1) we prove some of the Ostrowski type inequalities for functions of various classes and different types of measures. Among other results, we generalize some results of [1] as well as the recent results of [2].

References:

[1] Lj. Dedić, M. Matić, J. Pečarić, and A. Vukelić, *On generalizations of Ostrowski Inequality via Euler Harmonic Identities*, J. of Inequal. & Appl., **7,** 6 (2002), 787-805.

[2] Lj. Dedić, M. Matić, J. Pečarić and A. Aglić Aljinović, *On weighted Euler harmonic identities with applications*, Math. Inequal. & Appl., **8** (2), (2005), 237-257.

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