Montgomery's identities for function of two variables

(Talk)

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(joint work with Josip Pečarić)

Let $f:[a,b]\to\mathbf{R}$ be differentiable on $[a,b]$, and $f':[a,b]\to\mathbf{R}$ be integrable on $[a,b]$, then the following Montgomery identity holds,

$$f(x)=\frac{1}{b-a}\int_a^b f(t)\,dt+\int_a^b P(x,t)\,f'(t)\,dt$$

where $P(x,t)$ is the Peano kernel,

$$P(x,t)=\begin{cases}\frac{t-a}{b-a}, & a\le t\le x,\\ \frac{t-b}{b-a}, & x<t\le b.\end{cases}$$

Suppose now that $w:[a,b]\to[0,\infty\rangle$ is some probability density function, i.e. is a positive integrable function satisfying $\int_a^b w(t)\,dt=1$, and $W(t)=\int_a^t w(x)\,dx$ for $t\in[a,b]$, $W(t)=0$ for $t<a$ and $W(t)=1$ for $t>b$. The following identity is a generalization of Montgomery's identity,

$$f(x)=\int_a^b w(t)\,f(t)\,dt+\int_a^b P_w(x,t)\,f'(t)\,dt \tag{1}$$

where the weighted Peano kernel is

$$P_w(x,t)=\begin{cases}W(t), & a\le t\le x,\\ W(t)-1, & x<t\le b.\end{cases}$$

Using the result from *J. Pečarić, Some further remarks on the Ostrowski generalization of Čebyšev's inequality. J. Math. Anal. Appl. 123 (1) (1987), 18-33.*, we give weighted Montgomery's identities (1) for functions of two variables and further, obtain some new Ostrowski type inequalities for mappings of two independent variables. Also, we give some Grüss type inequalities for double weighted integrals.

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