Derivations, automorphisms and elementary operators

(Talk)

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Let R be a semiprime ring with the maximal right ring of quotients Q_{mr} and the symmetric Martindale ring of quotients Q_s.

An elementary operator (respectively, a generalized elementary operator) on R is a map $T\colon R \to R$ which can be expressed as a finite sum $T = \sum_{i=1}^{n} M_{a_i,b_i}$ of two-sided multiplication operators $M_{a_i,b_i}\colon x \mapsto a_i x b_i$ $(x \in R)$, with $a_i, b_i \in M(R)$ (respectively, $a_i, b_i \in Q_{mr}$).

A derivation on R is an additive map $d\colon R \to R$ satisfying $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. We say that d is inner (respectively, X-inner) if there exists $q \in M(R)$ (respectively, $q \in Q_s$) such that $d(x) = qx - xq$ for all $x \in R$. An automorphism $\phi\colon R \to R$ is said to be inner (respectively, X-inner) if there exists an invertible element $q \in M(R)$ (respectively, $q \in Q_s$) such that $\phi(x) = qxq^{-1}$ for all $x \in R$.

Motivated by the fact that elementary operators include both inner derivations and inner automorphisms, we consider derivations and automorphisms which are generalized elementary operators as well. We also give some counterexamples from the setting of C*-algebras.

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