Maximal Operands in Abacus Arithmetic

(Poster)

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The abacus is a well-known calculating tool with a limited number of placeholders for digits of operands, intermediate and final results. This implies that the four basic arithmetical operations are not performable with numbers with arbitrarily large numbers of digits. Given a number of rods n of the abacus (the most common value is $n = 13$) and a positive integer a (represented in an arbitrary basis B as a number with $\delta_B(a)$ digits), for all four arithmetic operation some natural questions arise: Can the operation be performed with a and any other operand b? If yes, what is the largest value $b_{\max}$ for b and is the operation feasible for a and any $b \leq b_{\max}$?

While the first question is relatively simple to answer for all four basic arithmetic operations, the second one proves non-trivial for multiplication and division. Namely, the value of $b_{\max}$ for multiplication and division on parity of n (for multiplication) and on the remainder of the division of n by 3 (for division). Even so, for division there are intervals of values for b (depending on n, a, B) in which the operation is not performable with a and all values of b. In this presentation a theorem on exact bounds for second operands shall be stated, the basic ideas of its proof given, and some implications for teaching arithmetic discussed.

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