Householder's approximants and continued fraction expansion of quadratic irrationals

(Poster)

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Let α be a quadratic irrational. It is well known that the continued fraction expansion of α is periodic. We observe Householder's approximant of order $m-1$ for the equation $(x-\alpha)(x-\alpha')=0$ and $x_0=p_n/q_n$: $R_n^{(m)}=\frac{\alpha(p_n/q_n-\alpha')^m-\alpha'(p_n/q_n-\alpha)^m}{(p_n/q_n-\alpha')^m-(p_n/q_n-\alpha)^m}$. We say that $R_n^{(m)}$ is good approximant if $R_n^{(m)}$ is a convergent of α. When period begins with a_1, there is a good approximant at the end of the period, and when period is palindromic and has even length ℓ, there is a good approximant in the half of the period. So when $\ell\le 2$, then every approximant is good, and then it holds $R_n^{(m)}=\frac{p_{m(n+1)-1}}{q_{m(n+1)-1}}$ for all $n\ge 0$. We prove that to be a good approximant is the palindromic and the periodic property. Further, we define the numbers $j^{(m)}=j^{(m)}(\alpha,n)$ by $R_n^{(m)}=\frac{p_{m(n+1)-1+2j}}{q_{m(n+1)-1+2j}}$ if $R_n^{(m)}$ is a good approximant. We prove that $|j^{(m)}|$ is unbounded by constructing an explicit family of quadratic irrationals, which involves the Fibonacci numbers.

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